Let's Build an Electric Diddley-Bo Guitar!

Music is Math!!

Pitch and Frequency

- Notes on a keyboard or a guitar are pitches
- Sound is the vibration of air between us and whatever is making the sound
- Different pitches have different frequencies...
- Frequency is how many vibrations happen in a certain amount of time
- For music we measure frequency in Hertz (Hz) which means number of vibrations per second

But how does pitch relate to frequency?

For every Octave in pitch from Do to Do or Re to Re we actually double the frequency

For the note A above middle C or A_4 the frequency is 440Hz
for A_5, an octave up, the frequency is 880 Hz

See if you can fill in this table...

Note (pitch)	Frequency
A_3	..... Hz
A_4	440 Hz
A_5	880 Hz
A_6	..... Hz

Harmony

If I play A4 and A5 together, every time the A4 string vibrates the higher A5 string vibrates exactly twice.

If I go up another octave to A6, every time my A4 string vibrates my A6 vibrates 4 times. In other words, four times the frequency of A4.

What if I want my string to vibrate at 3 times the frequency of A4? What note is that?

Well that would be F5 which is the **Third Harmonic** of A4 and a **Perfect Fifth** (or 5 whole notes) above A5!

For every 2 vibrations of my A5 string I get exactly 3 vibrations of my D5 string which is a perfect fifth above it! A frequency ratio of 3:2

Notes that have frequencies with nice relationships like this vibrate in **Harmony**... and we like it when that happens!

Pitch Interval	Frequency ratio
Octave	2:1
Perfect Fifth (Eg. D_5 & A_5)	3:2
Perfect Fourth (Eg C_5 & A_5)	4:3
Major Third	5:4
And so on...	... and so forth

Tuning Systems

Most instruments we play nowadays break up each octave into 12 evenly spaced pitches called semitones.

This is called 12 tone equal temperament tuning and it's what modern western music uses.
Breaking up the octave like this lets us get pretty close to our favourite harmonic intervals, but also lets us change the root note, change key, or do things like the circle of fifths.

To calculate the **frequency** of the next semitone up we use this equation:

$$f \times \sqrt[12]{2}$$

OR

$$f \times 2^{\frac{n}{12}}$$

f = frequency of current note

n=number of semitones above current note

Before this scale we used Just Intonation which uses those nice perfect ratios but doesn't let us change key without going out of tune.

Name	**C**	**D**	**E**	**F**	**G**	**A**	**B**	**C**
Ratio	1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

Frequency and Wavelength or String Length

Wavelength is the space between each vibration that reaches our ear.
Wavelength is **inversely proportional** to frequency....

$$Wavelength \equiv \frac{1}{Frequency}$$

Put another way, every time we go up an octave and double the frequency we half the wavelength.

On a guitar or piano the wavelength is **directly proportional** to the string length

So on a guitar, to go up one octave we half the string length using a fret!

We add more frets in-between to get all 12 semitones with frequencies that fit 12 tone equal temperament tuning.

Calculating and drawing frets

People who build guitars (luthiers) use a special rule to work out where to put the next fret

1. They work out how long the string is at the **current fret**
2. They divide this number by **17.8**
3. **The next fret is this far from the current fret**

Lets try it for the first fret

Fret Number	Length of string At this fret	÷ 17.8 =___	Length of string At next fret
Fret 0 (nut)	712 mm	40 mm	672 mm
Fret 1	672 mm	37.75 mm	...
Fret 2			
Fret 3			
Fret 4			
Fret 5			
Fret 6			
Fret 7			
Fret 8			
Fret 9			
Fret 10			
Fret 11			
Fret 12			

Challenge:
How did the luthiers come up with the number 17.8??
Can you derive this number from the
Hint: replace frequency with $\frac{1}{String\ Length}$

Let's try it for Just Intonation

Note Number	**Length of string At this fret**	Multiply by this fraction	Length of string at this note
1 (nut)	712 mm	1/1	712 mm (no change)
2	712 mm	8/9	639 mm
3	712 mm	4/5	
4	712 mm	3/4	
5	712 mm	2/3	
6	712 mm	3/5	
7	712 mm	8/15	
8 (octave)	712 mm	1/2	